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ABSTRACT

This report describes a classroom observation study in Korean primary mathematics classrooms. What was discovered in Korean classrooms was children, through higher level questioning, were being led to conceptualize the constructs and operations. The textbook was focused on the concepts in a concise, coherent and systematic manner similar to what was observed in the classroom. This report includes sample translation of a textbook chapter used in one of the lessons. (KHR)

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Conceptualization of the Constructs in Korean Primary Mathematics

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Conceptualization of the Constructs in Korean Primary Mathematics

Grow-Maienza was invited to fill an exchange professorship at Pusan National University in South Korea just as the results of the TIMSS were coming out ranking Korean students second in the world to Singapore (and the United States students 28th in 41 nations tested),. Grow-Maienza took the opportunity to replicate some of the work Harold Stevenson and his group at the University of Michigan had done in Japan, Taiwan, and China in the '80's and '90's in primary mathematics classrooms using the Michigan instruments. With Hahn and Joo in Pusan, Grow-Maienza conducted the first classroom observation study in Korean primary mathematics classrooms, and published the results in the *Journal of Educational Psychology in June, 2001.*

What was discovered in Korean classrooms was children, through higher level questioning, were being led to conceptualize the constructs and operations. Informed by the graduate student observers that teachers were staying very close to the textbook, we translated a textbook chapter used in one of the lessons. We found the textbook to be focused on the concepts in a concise, coherent and systematic manner similar to what was observed in the classroom.

Through Truman Faculty Research grants Eisenhower Foundation funds and largely through National Science Foundation Award #0086580 most of the series has been translated. Analysis of the series can be accessed in the report to the NSF at: http://eisenhowermathematics.truman.edu .

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Here are some highlights about *Korean Mathematics*.

Highlights of Differences in *Korean Mathematics*

- Focus on conceptualization of just a few constructs in 5 domains you will recognize:
- Block structure than can be easily accessed and integrated into other curricula, reform curricula or traditional curricula
- Coherence that comes from at least 2 sources:
    1. The lessons tell a story, have a beginning, a middle, and an end, and move from simple and concrete to more and more complex.
    2. Connections are made in the lessons to the deep meanings of fundamental mathematics.
        a. *Korean Mathematics* focuses from the beginning on the inverse relationship of addition/subtraction, and of multiplication/division.
        b. *Korean Mathematics* focuses always on the decimal, base 10 system, so that children from the beginning understand place value.
- All concepts in *Korean Mathematics* are presented in multiple models, and multiple modes.

Korean Mathematics is focused on just a few constructs in 5 domains that is easily recognized.

Domains in *Korean Mathematics* compared to NCTM Content Strands

Content Strands in NCTM (2000)	Domains in *Korean Mathematics*
Number and Operations	Numbers
Algebra	Operations
Geometry	Figures
Measurement	Measurement
Data Analysis and Probability	Relations

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Korean Mathematics is focused on a block structure that can be easily accessed and integrated into other curricula—reform curricula or traditional curricula.

6-1 | Mixed multiplication of fractions by decimals

5-2 | Multiplication of 3 decimals
Decimal x decimal
Position of decimal point
Mixed decimal x natural number
Decimal x Natural Number

5-1 | Multiplication of fractions
Relation between fractions and decimals

4-2 | Multiplication
4-1 | Places of decimals

3-2 | Introduction of decimals
3-1 | Application of multiplication
Mental multiplication of 2-place number x 1-place number
Multiplication of 2-place number x 1-place number

2-2 | Division using multiplication table
Multiplication table
2-1 | Basics of multiplication

1-2 | Counting by 2's, 3's, 4's, 5's
Numbers increased by 2, 5, 3, or 4
Counting by jumping 2, 5, 3, or 4
Number of groups and total sum
Number in a group and number of groups

1-1 | Numbers

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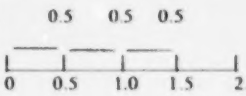
In the teachers' manuals and student texts, in general, each instructional sequence follows this sequence:

- Have students in groups discover how we might use the concept studied
- Present a word problem
- Explain the problem on a number line
- Formalize the problem into a sentence, then into an algorithm
- Calculate vertically, and
- Then solve more word problems.

Multiplication of Decimals

Let's study multiplication of decimals and natural numbers. *Emily drinks 0.5 liters of milk per day. How many liters would she drink in 3 days?*

How can we calculate 0.5 x 3 ?



On the number line, we see 0.5 x 3 = 1.5

or

$0.5 \times 3 = \frac{5}{10} \times 3 = \frac{5 \times 3}{10} = \frac{15}{10} = 1.5$

Like the example above, we change the decimal to a fraction and multiply natural numbers, and change the fraction to a decimal.

$0.3 \times 7 = \frac{\square}{10} \times 7 = \frac{\square}{10} =$

From *Korean Mathematics*, Book 5-2-1, p. 8

Coherence comes from at least two sources. First, the lessons tell a story - have a beginning, a middle and an end. Each instructional sequence goes from the simple and concrete to more complex. For instance, in these lessons on multiplication of decimals in one of four chapters in the two 5th grade texts the topics in the instructional sequences are formatted the same and each sequence using the same format becomes more and more complex.

Decimal x Natural Number	0.5 x 3 = 1.5
Mixed decimal x natural number	3.15 x 12 =
Position of decimal point	$9 \times 0.004 = 9 \times \frac{4}{1000} = \frac{9 \times 4}{1000} = \frac{36}{1000} = 0.036$
Decimal x decimal	0.5 x 0.09
Multiplication of 3 decimals	1.2 x 0.3 x 4.21

Secondly, coherence in *Korean Mathematics* comes from the connections to the deep meanings within mathematics. Generalizations made in the report to the NSF about *Korean Mathematics* point up the connections that mark the series with coherence, and clear, in-depth conceptualizations for every construct in the series. Three of those connections illustrated here are: focus on the inverse relationships in mathematics, focus on the base 10 decimal system in mathematics, and multiple models and strategies in every lesson

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Focus on the Inverse Relationships in Mathematics

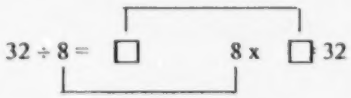
All operations in *Korean Mathematics* are systematically and coherently linked to the inverse relationships of addition & subtraction, and of multiplication & division.

Addition and Subtraction

From the very beginning of study on number, addition and subtraction are not separated but taught from as the inverse of one another. That is, in the very first number lesson in Book 1-1, objects are put together and taken apart to demonstrate that 2 & 1 put together are the same number as 3, and that 3 separated into 2 & 1 are the same number.

Instruction on the operations of addition and subtraction begin in Grade one with one chapter titled "Merging and separating number", in Book 1-1, and four chapters named "Addition and Subtraction," two in Book 1-1, two in Book 1-2. The concepts of putting together, taking apart, and even comparing two sets to see how many more, are treated together, alternately one after the other.

Multiplication and Division



The inverse relationship between multiplication and division is treated in Grade 2 in *Korean Mathematics*, with:

1) an array,

How many ?

5 x 3 = 15 *15 ÷ 3 = 5*

2) a two-part word problem focusing on division of 15 by the two multiples illustrated in the array above,

If 15 are bundled by 5, how many bundles?
15 ÷ 5 = □ *□ bundles*

If 15 is divided by 3 bundles, how many in a bundle?
15 ÷ 3 = □ *□*

and

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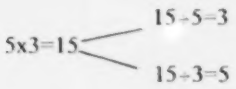
3) a diagram demonstrating that the multiplication fact can be written as 2 division expressions.

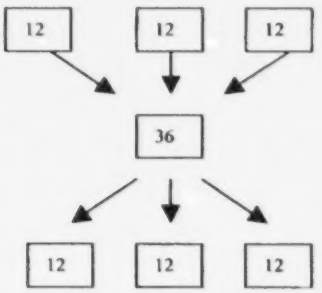


From *Korean Mathematics*, Book 2-2-6

The relation between multiplication and division is conceptualized in Grade 3 with a two-part problem and a very explicit diagram showing the inverse relationship.

There are 12 pencils in one dozen pencils.
How many pencils in 3 dozen? 12 x 3 = 36

If divided among 3 persons., how many pencils go to each person?
36 ÷ 3 = 12



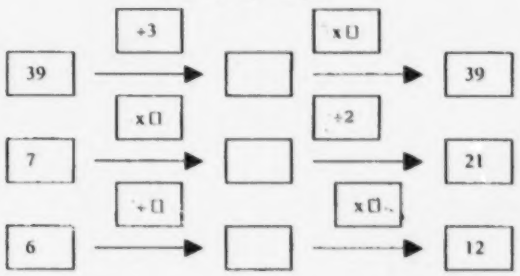
12 x 3 = 36

36 ÷ 3 = 12

From *Korean Mathematics*, Book 3-1-6, p. 80

and with this function machine:

Fill in the boxes.



From *Korean Mathematics*, Book 3-1-6, p. 87

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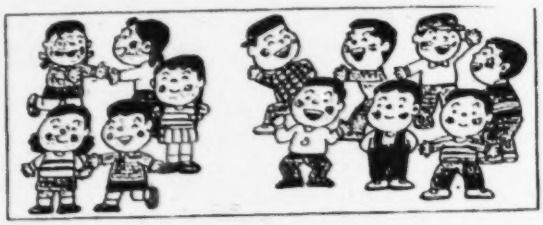
Focus on the Base 10 Decimal System in Mathematics

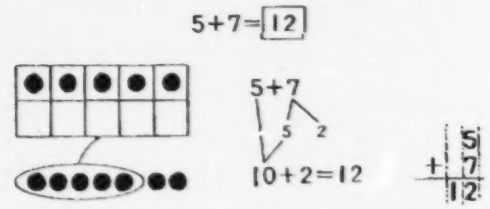
Addition and Subtraction

From the beginning study of number, addition and subtraction are taught focused on the decimal place value system. Children are not expected to learn facts beyond sums of ten. Children are first taught numbers that sum to 5, then those that add to 10. Then, if adding 6 + 8, for instance, children base their responses on knowledge of sums of 10, separate the smaller addend (6) into 4 + 2, add 2 to 8 to make 10, and then add the remaining portion of 6 (4) to 10 to make 14.

Chapter 4 in Book 1-2 models in four different ways on one page the expression 5 + 7 = 12. First a picture of 12 children in groups of 5 and 7 which can be counted is shown, then the horizontal expression. The separation of 7 into 5 and 2 and the addition

There are 5 girls and 7 boys in the playground.
How many children are there?





***Korean Mathematics,* Book 1-2-4, p. 46**

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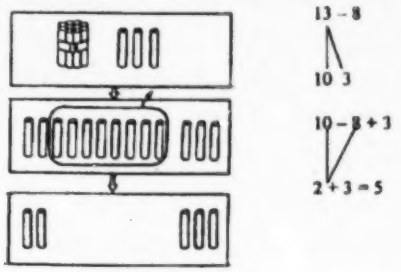
Two ways of subtracting a one digit number from a number up to 19 with renaming are also taught in *Korean Mathematics*.

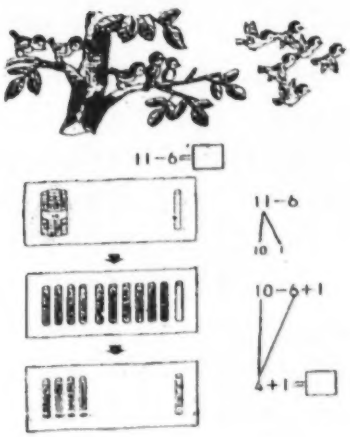
The Subtraction-Addition Method



From *Korean Mathematics*, Teacher Manual 1-2-4, p. 89

The student textbook models the concept with a word problem, the expression, a picture of a bundle of tens and one and a number diagram demonstrating the separation of the 11 into a ten and a one, as seen below.

There were 11 robins.
6 robins flew away.
How many robins are there now?



From *Korean Mathematics*, Book 1-2-4, p. 49

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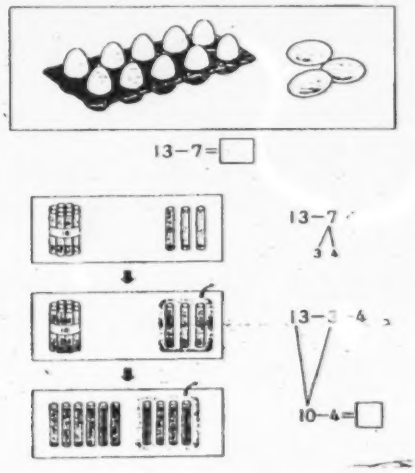
The Subtraction-Subtraction Method

There are 13 eggs. 7 of the eggs are taken away. How many eggs are there now?



From *Korean Mathematics*, Book 1-2-4, p. 51

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Multiple Models and Strategies in every Lesson

Multiple ways of modeling and multiple strategies are seen throughout the curriculum in *Korean Mathematics*. Major objectives in the teacher manuals are two: 1) understanding the principle, and 2) calculating the procedure. Each lesson models the problems with stories, pictures, number lines, diagrams, number sentences, and formats each operation both vertically and horizontally. This is one lesson observed by Grow-Maienza, Hahn and Joo in Pusan (2001).

Verbal: Yun-Ho has 24 pieces of paper and Young-Mi has 43 pieces of paper. How many pieces do they have?

Numbers and Fingers: 24 + 43 = on the felt board.
In response to teacher's leading questions as to how many tens and ones there were, children held up 6, then 7 fingers.

T. directed students with LQs to do the problem on desks with counters.

Concrete Manipulatives:

Numbers in Horizontal Format

T. demonstrated addition of tens and ones on the blackboard thus, first adding the tens, then the single units:

First, the tens: $20 + 40 = 60$

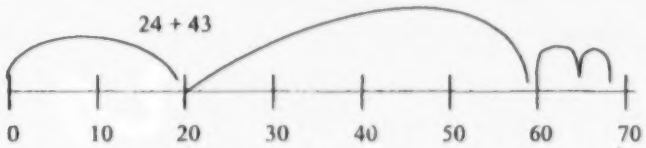
$24 + 43 =$

Then, the ones: $4 + 3 = 7$

and then explained the tens and ones places.

Number line:

The same problem was presented on a number line:



Vertical Format:

Finally the problem was presented in a vertical format:

$$\begin{array}{r} 24 \\ +43 \\ \hline \end{array}$$

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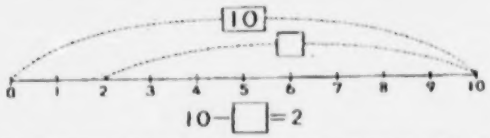
Number Lines

Number lines are ubiquitous in the Korean curriculum. The number line is used to conceptualize everything linear, including the concept of time. Below are some examples of how the number line is modeled across grade levels and domains in *Korean Mathematics*.

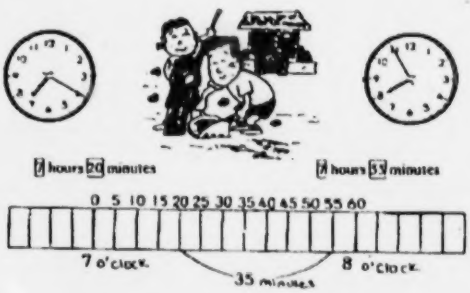


"Take away" subtraction modeled on the number line (Book 1-2-1, p. 9)

Rabbit has gone to the right from 0 how many places?
Turtle has gone to the right from 0 how many places?
How many more places did the rabbit go than the turtle?

"Comparison" subtraction modeled on the number line (Book 1-1-5, p. 65)

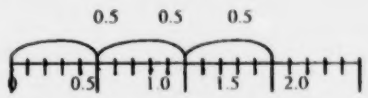
the time of starting cleaning **the time of finishing cleaning**



"Time" modeled on the number line (Book 2-1-6, p. 62)

Emily drinks 0.5 liters of milk per day.
How many liters would she drink in 3 days?

How can we calculate 0.5 x 3 ?



Multiplication of a decimal with a natural number modeled on the number line (Book 5-2-1, p. 8)

Stanford is making two animal shapes by dividing in half a wire 5m in length.

Let's find out the length of the wire for one animal shape.
How can we do the division, 5 ÷ 2 ?

Division of a number into fractions modeled on the number line (Book 5-2-2, p. 26)

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